

~~maXbox~~



# maXbox Starter 41

## Deal with Big Numbers

### 1.1 A Big Decimal or Big Int Interface

Today we step through numbers and infinity.

As you may know there's no simple solution to print, calculate or store big numbers or decimals, for example you want to compute  $400000078669 / 2000123$  your calculator shows (so does my Casio FX-880P):

199987.7401

So this is not the end of the line, a second test is

```
maxcalcF('400000078669 / 2000123')
```

and we get: 199987.740088485

And there are even more numbers that need to compute so we switch to <http://www.wolframalpha.com> to get the real precision thing or at least an approximation:

199987.7400884845581996707202507045816682274040146530988344  
21683066491410778237138415987416773868407092963782727362267  
220...

<http://www.wolframalpha.com/input/?i=400000078669%2F2000123>

again as you suppose the numbers go on.

Use "Power Towers" to write them down. The decimal point is the most important part of a decimal number like above. Without it, we would be lost ... and not know what each position meant.

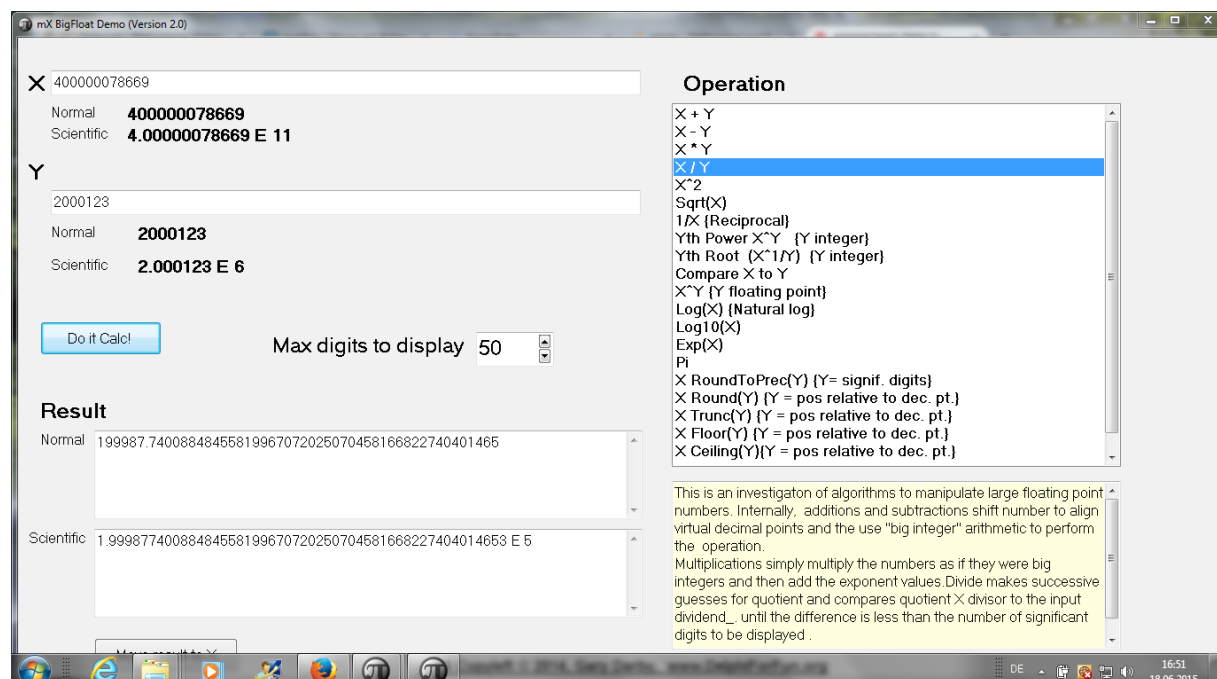
Dividing decimals is almost the same as dividing whole numbers, except you use the position of the decimal point in the dividend to determine the decimal places in the result. Our division is always an approximation.

Approximate means you're going to round the number. Because you're not actually giving the exact number, all those numbers after the decimal, the rounded number is called an approximation:

199987.**7401** is roundToPrec4 of: 199987.740088485

Although, you probably wondered how they get those nice and fancy graphical user interfaces (GUI) for large numbers, here in maXbox we do also have one:

maXbox3 568\_U\_BigFloatTestscript2.pas Compiled done: 6/18/2015



The idea that you are approximating is that, as you are only taking the first 50 decimal places as you can see at the screen-shot. The same like wolfram goes like this:

199987.7400884845581996707202507045816682274040146530988344  
21683066491410778237138415987416773868407092963782727362267  
2205

When we try to write this decimal number (or the well known PI or SQR(2)) in decimal notation, we get an endless stream of digits.

3.141592653589723.....and so on forever.

But suppose instead, we use fractional notation. Then we can write each part as a precise (irreducible)

$$\frac{400000078669}{2000123}$$

A fraction is an exact ratio of 2 numbers, and if those 2 numbers are integers, or at least rational numbers, then the fraction can more

appropriately be called a rational number. An irrational number can be represented as an approximation to a rational number to an extremely high degree of accuracy.

It's quite clear that there are fractions which can't be expressed in finite decimal form!

Now, here's the **big** problem. Not every number is rational! For example there is no fraction for  $\sqrt{2}$ . That is, no matter what whole numbers  $m$  and  $n$  you pick,  $m/n$  is not the square root of 2. Euclid wrote down a real AND beautiful proof of this fact around 2300 years ago.

Interesting point about those real numbers is also the possibility to divide the number to his prime factorization:

$29 \times 37 \times 127^{(-1)} \times 179 \times 15749^{(-1)} \times 2082607$

**maxcalcF**('29\*37\*(127^-1)\*179\*(15749^-1)\*2082607');

>> 199987.740088485

## 1.2 Real Big Integer

So what about big integers? For example you want to compute  $\text{fact}(70)$ , your calculator shows:

$\text{fact}(70) = 1.19785716699699\text{e}+100$  or **maxcalcF**('70!')

1.19785716699699E100

or even more

1.1978571669969891796072783721689098736458938142546425857...  
 $\times 10^{100}$

but the maximum range on Pascal, C or Delphi depends on your operating system types, means nowadays an  $\text{int64}$  range is big.

Now that the "signed" words are finally up-to-par with the unsigned integer types, languages introduce a new 64-bits integer type, called  $\text{Int64}$ , with a whopping range of  $-2^{63}..2^{63} - 1$

Another way is to use a type extended, but the limitation is precision like

```
Writeln(FloatToStr(Fact(70)))
```

it only shows 1.2E+0100 or 1.19785716699698966E100

With a  $\text{BigInt}$  Library you'll see the full range of  $\text{Fact}(70)$ :

11978571669969891796072783721987892755536628009582789845319  
680000000000000000

All examples can be found online:

maxbox3\examples\161\_bigint\_class\_maxprove2.txt

[http://www.softwareschule.ch/examples/161\\_bigint\\_class\\_maxprove2.txt](http://www.softwareschule.ch/examples/161_bigint_class_maxprove2.txt)

The call respectively the calculation goes like this:

```
function GetBigIntFact(aval: byte): string;  
//call of unit mybigint  
var mbRes: TMyBigInt;  
    i: integer;  
begin  
    mbRes:= TMyBigInt.Create(1);  
    try  
        //multiplication of factor  
        for i:= 1 to aval do  
            mbRes.Multiply1(mbres, i);  
        Result:= mbRes.ToString;  
    finally  
        //FreeAndNil(mbResult);  
        mbRes.Free;  
    end;  
end;
```

Or you want the power of 100 like  $2^{100}=12676506002282299670376$

```
function BigPow(aone, atwo: integer): string;  
var tbig1, tbig2: TInteger;  
begin  
    tbig1:= TInteger.create(aone);  
    //tbig2:= TInteger.create(10);  
    try  
        tbig1.pow(atwo);  
    finally  
        result:= tbig1.toString(false);  
        tbig1.Free;  
    end;  
end;
```

At least one really big, it's **333<sup>4096</sup>** (10332 decimal digits)!:

85424105895770887322966965917914584710138161386222147182917  
67781049536057906627318361093758865620577697322240787369539  
81504332246815140327668478948527046875787550310970504170251

82159123158149832522632506558096888465749900859669714028055  
71722261672173081615765772729999913389932448720516828003070  
67049948583875478861163177472370389199969581395362347697256  
70960895462872160261732690973238984844130761428601643232816  
96899530174488741933479651249141554621396684104620616204908  
08738502674650541288448161671070156327238251274328179200400  
93890996761080835353394335725524764503872061091991652109449  
19392226034862115885018213075725104899320094825429972785833  
08567702799428942416066175248810727171285940075837283674910  
40238020148038165962562077292113243073436214316216978130335  
33870396651279893801306533507055182475982690042018637954924  
71837910714543445430478178723995293717173465603902499439767  
52390512439014099231872228674262774255372493330431055176825  
66636232366673489784851223121246465215815916502622714756503  
41470134146173462494457636856525872971987877831270158832603  
81720501132263714345569154398039964038547327865662026993558  
29785045595252413682840639314284074867518100746598657655945  
81378019251453464202570850546505546651918600042626257608188  
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43720126167236379124481127893994316258310107716760682781353  
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36501117701055868582355811954302737246702288220571315639590

```

61822027640052204288908482254041966887584978480948804804531
68842877575361652520057158629595160885099803962695340286603
65420928832906521186092765576521399427875156924799554483049
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13511381906756017382397217776898779213454899732222634652574
89935099549137644474027777583956137051269588783021487059246
17598489865075863183194186708153376785925896453521253497876
13552701597508011611524503843754837913945812148834125113809
99171425821993170789973409296543662081

```

I'm trying to move a part of SysTools to Win64. There is a class `TStDecimal` which is a fixed-point value with a total of 38 significant digits. The class itself uses a lot of ASM code.

```

function BigDecimal(aone: float; atwo: integer): string;
begin
  with TStDecimal.create do begin
    try
      //assignfromint(aone)
      assignfromfloat(aone) //2
      RaiseToPower(atwo) //23
      result:= asstring
    finally
      free
    end;
  end;
end;

```

SysTools is hosted under Sourceforge:

<http://www.sourceforge.net/projects/tpsystools>

The class `TStDecimal` is defined in the unit `StDecMth`. It has the following description: `StDecMth` declares and implements `TStDecimal`. This is a fixed- point value with a total of 38 significant digits of which 16 are to the right of the decimal point.

```
1366556882568704.2292943165706246
```



☝ It is important to note that Infinity is not a real number, it is an idea. An idea of something without an end.

Infinity is not "getting larger", it is already fully formed. Sometimes students or people (including me) say it "goes on and on" which sounds like it is growing somehow. But infinity does not do anything, it just is.

```
Writeln("")
Writeln("Big Lotto Combination 1600 of 5000!")
Writeln("")
Writeln(BinominalCoefficient(5000, 1600));
Writeln("")
Writeln("Same Lotto Comb 6 of 45!")
Writeln(BinominalCoefficient(45, 6)); -->8145060
Writeln("Same Lotto Comb 39 of 45!")
Writeln(BinominalCoefficient(45, 39)); -->8145060
```

OK,  $1/3$  is a finite number (it is not infinite). But written as a decimal number the digit 3 repeats forever (we say "0.3 repeating"):

0.33333333333333333333... (etc.)

There's no reason why the 3s should ever stop: they repeat infinitely. Okay, I hope you're not one of those people who denies  $0.999... = 1$ , because it sounds like you're saying 0.333... doesn't exactly equal  $1/3$ . If there are only a finite amount of 3s, I wouldn't argue a bit that they're not equal, but with an infinite amount, they are.

Test the script with **F9** / F2 or press Compile.

**Conclusion** And we can easily create much larger numbers than those! But none of these numbers are even close to infinity. Because they are finite, and infinity is ... not finite!

"Wise men speak because they have something to say; Fools, because they have to say something". -Plato

Feedback @ [max@kleiner.com](mailto:max@kleiner.com)

Literature: Kleiner et al., Patterns konkret, 2003, Software & Support

[http://www.softwareschule.ch/download/codesign\\_2015.pdf](http://www.softwareschule.ch/download/codesign_2015.pdf)

[http://www.softwareschule.ch/download/XXL\\_BigInt\\_Tutorial.pdf](http://www.softwareschule.ch/download/XXL_BigInt_Tutorial.pdf)

<http://www.mathsisfun.com/numbers/infinity.html>

<https://github.com/maxkleiner/maXbox3/releases>

### 1.3 Appendix Study with BigInt Direct

*// TODO: Copy a file in a connected share path  
//this is 333^4096:*

```
function GetBigIntDirect: string;  
    //Unit mybigint  
var mbResult: TMyBigInt;  
    i: integer;  
begin  
    mbResult := TMyBigInt.Create(333);  
    try  
        // Faktoren im Zaehler aufmultiplizieren ---> 2^12=4096  
        for i:= 1 to 12 do begin  
            mbResult.Multiply(mbresult, mbresult);  
            //writeln(inttostr(i)+'': '+mbresult.toString);  
        end;  
        Result := mbResult.ToString;  
    finally  
        //FreeAndNil(mbResult);  
        mbResult.Free;  
    end;  
end;
```

```
TMyBigInt = class  
    private  
        Len: Integer;  
        Value: AnsiString;  
        procedure Trim;  
        procedure Shift(k: Integer);  
        procedure MultiplyAtom(Multiplier1: TMyBigInt; Multiplier2: Integer);  
    public  
        constructor Create(iValue: Integer = 0);  
        procedure Add(Addend1, Addend2: TMyBigInt);  
        procedure Multiply(Multiplier1, Multiplier2: TMyBigInt); overload;  
        procedure Multiply(Multiplier1: TMyBigInt; Multiplier2: Integer); overload;  
        function ToString: string;  
        procedure CopyFrom(mbCopy: TMyBigInt);  
end;
```